

Mathematical modelling of eye condition in glaucoma: Approaches to parameter analysis and their interactions

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Abstract. Mathematical modelling of physiological processes is a key component of intelligent medical systems, as it describes disease mechanisms in greater detail and contributes to early diagnosis. This study presents an analytical model for assessing eye health, incorporating key ophthalmological parameters: intraocular pressure (IOP), perfusion coefficient (Pperf), best-corrected visual acuity (BCVA), visual field index (VFI), retinal nerve fibre layer thickness (RNFL), and neuroretinal rim area (Rim_area). The study aimed to develop a model that can accurately evaluate the nonlinear interactions between these parameters, improving diagnostic accuracy and predicting glaucoma progression. The study also aimed to determine critical threshold values of these ophthalmological indicators to improve clinical decision-making. The results demonstrated that application of numerical optimisation techniques such as L-BFGS-B and logarithmic-exponential transformations significantly improves the accuracy of glaucoma risk prediction; critical threshold values of ophthalmological parameters have been identified, improving precision of detection of glaucoma stages. Additionally, the study facilitates a systematic evaluation of the association between intraocular pressure and optic nerve condition, a factor deemed critical for accurate prediction of disease progression. The practical significance of this research is determined by the potential integration into medical IT systems for automated glaucoma screening and patient monitoring. The proposed approach can assist ophthalmologists in clinical decision-making by optimising treatment strategies and preventing irreversible vision loss. The model's adaptability also enables its use in telemedicine applications, facilitating remote diagnostics and continuous patient assessment

Keywords: analytical model; ophthalmological parameters; optimisation; medical diagnostics; adaptability

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Introduction

Mathematical modelling is crucial in medicine, particularly in analysing complex biological processes and predicting disease progression. In ophthalmology, the development of predictive models is highly relevant for diagnosis and management of glaucoma, a chronic disease that remains one of the leading causes of irreversible blindness. Open-angle glaucoma, the most prevalent form, is characterised by progressive optic nerve damage, which is correlated with impaired aqueous humour dynamics and changes in the biomechanical properties of ocular tissues. Current research in mathematical modelling of glaucoma emphasises three primary directions: physical and biomechanical modelling of intraocular pressure (IOP) regulation and its effect on ocular structures; statistical approaches for risk factor analysis and disease progression prediction; artificial intelligence (AI) methods for automated diagnostics and patient-specific treatment optimisation.

A comprehensive comparison of traditional machine learning algorithms was conducted by Y. Tong *et al.* (2020), evaluating the effectiveness of Support Vector Machines (SVM) and Random Forest algorithms in automated glaucoma diagnosis. The findings provided an initial benchmark for AI applications in ophthalmology. However, the study did not incorporate transformer-based models, which can significantly improve medical image analysis. I. Wagner *et al.* (2022) reviewed modern glaucoma management strategies, emphasising the role of advanced diagnostic tools, including Optical Coherence Tomography (OCT) and automated perimetry. The study also highlighted the advantages of Minimally Invasive Glaucoma Surgery (MIGS) over traditional trabeculectomy in reducing postoperative complications. Despite these insights, the review did not fully explore AI-driven diagnostic approaches, which have emerged as a key area of research. Y. Liu *et al.* (2023) introduced an innovative approach by integrating mathematical modelling with clinical data to predict glaucoma progression based on IOP fluctuations and visual field deterioration. Their algorithms demonstrated high predictive accuracy but lacked extensive validation in large clinical cohorts, limiting their immediate applicability in real-world settings. A meta-analysis by T. Dube *et al.* (2023) addressed deep learning-based glaucoma detection techniques, particularly convolutional neural networks (CNNs) such as ResNet. The study reported over 95% sensitivity in fundus image classification. However, inconsistencies in dataset standardisation among different studies complicated the determination of an optimal model architecture. A. Shoukat *et al.* (2023) achieved state-of-the-art results in optic disc segmentation using transfer learning techniques and demonstrated the adaptability of AI models in ophthalmic image processing. However, the “black-box” nature of deep learning algorithms posed challenges in clinical interpretation and trust among medical professionals. R. Kashyap *et al.* (2022) improved the U-Net architecture for segmentation tasks, significantly improving precision

on REFUGE datasets. While their model outperformed previous techniques, its reliance on high-performance computing resources, particularly GPUs, presented limitations for deployment in resource-constrained medical environments. V. Vychuzhanin *et al.* (2024) analysed neovascular glaucoma and use of AI in analysis of angiographic patterns, achieving early detection of high-risk cases. Despite their promising results, the study was constrained by a small sample size ($n < 200$) and lacked comparative analysis with standard diagnostic protocols, raising concerns about generalisability. Y. Jin *et al.* (2024) provided a detailed review of AI architectures such as U-Net and Generative Adversarial Networks (GANs) for glaucoma detection. The study reported accuracy rates exceeding 94% in OCT analysis and noted the risk of overfitting due to the limited size of training datasets. Furthermore, the study did not propose practical guidelines for integrating AI-driven diagnostic systems into clinical workflows. S. Hussain *et al.* (2023) developed a Long Short-Term Memory (LSTM)-based model for predicting glaucoma progression, achieving an Area Under the Curve (AUC) of 0.92 using longitudinal IOP data. While the study highlighted the potential of AI in prognostic modelling, the retrospective nature of their data introduced selection bias, underscoring the need for prospective trials. T. Moudgil & D. Gupta (2024) conducted an extensive review of glaucoma treatment options, confirming the effectiveness of Selective Laser Trabeculectomy (SLT) and MIGS as safer alternatives to traditional surgical interventions. However, their study lacked a cost-benefit analysis, which is crucial for healthcare policymakers in evaluating the economic feasibility of these procedures.

Despite significant progress in mathematical and AI-based modelling of glaucoma, several key gaps remain in the existing literature. Firstly, many studies do not adequately integrate nonlinear interactions between ophthalmic parameters, such as corneal hysteresis, retinal nerve fibre layer thickness, and visual field deterioration. Second, while deep learning has demonstrated high accuracy in diagnostic tasks, issues of interpretability and generalisation across diverse patient populations persist. Third, current predictive models often fail to incorporate uncertainty quantification, limiting their use in clinical applications. Lastly, the translation of AI-based diagnostics into routine ophthalmic practice remains challenging due to the lack of standardised datasets and regulatory frameworks. The study aimed to address these gaps by developing an optimised mathematical model that combines elements of statistical analysis and AI-driven prediction. By incorporating parameter normalisation techniques and accounting for nonlinear dependencies, the proposed approach seeks to improve diagnostic precision and enhance the early detection of glaucoma progression. Furthermore, this research explored the feasibility of integrating AI models into clinical decision-making, ensuring their practical applicability in ophthalmology.

Materials and Methods

Mathematical model of eye condition in glaucoma

This section presents the approach to modelling the condition of the eye in primary open-angle glaucoma using selected physiological and diagnostic parameters. The methodology was based on the analysis of the impact of these parameters on disease progression and the functional state of the visual system. The justification for selection of indicators, crucial for the accuracy and clinical relevance of the model, was emphasised. The selection of key parameters for eye condition modelling was based on physiological and diagnostic factors that characterise the state of the eye in primary open-angle glaucoma. These parameters were selected for their influence on glaucoma progression and their relationship with the functional status of the eye, incorporating the anatomy of the eye (Fig. 1).

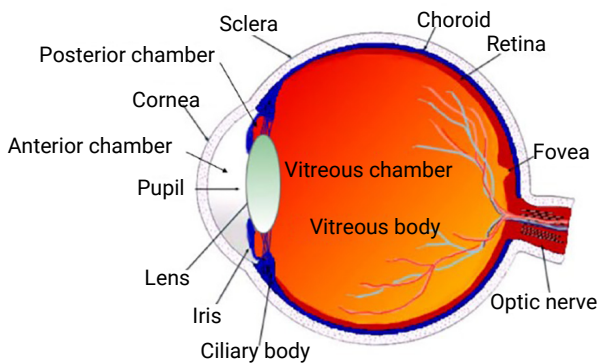


Figure 1. Anatomy of the eye

Source: compiled by the authors

The following parameters were selected: intraocular pressure (IOP) – elevated IOP, the primary risk factor for glaucoma development (Harris et al., 2013); risk coefficient (RQ) – volume intraocular blood flow coefficient (RQ), deteriorating intraocular blood flow reduces oxygen and nutrient saturation to the retina and optic nerve, which contributes to the development/progression of glaucoma

(Tonti et al., 2024); best-corrected visual acuity (BCVA) – BCVA evaluation, crucial for determining functional vision impairments associated with glaucoma progression (Tarcoveanu et al., 2022); visual field index (VFI) – VFI, used to quantitatively assess visual field loss and monitor disease progression (Tarcoveanu et al., 2022); perfusion pressure (Pperf) – perfusion pressure disturbances can affect optic nerve blood supply, contributing to glaucoma development (Siesky et al., 2023); anterior chamber angle (alpha_t1) – can be used to diagnose different forms of glaucoma, including open-angle and closed-angle types (Tonti et al., 2024); age – the risk of developing glaucoma increases with age, as confirmed by numerous studies (Jin et al., 2024); thickness of ganglion cell layer with inner plexiform layer (GCL_IPL) – the reduction in thickness of this layer in glaucoma is associated with degeneration of optic nerve fibres in the central zone of the retina (Tonti et al., 2024); retinal peripapillary nerve fibre layer (RNFL) thickness – a decrease in RNFL thickness is a marker of glaucoma progression (Tonti et al., 2024); neuroretinal rim area (Rim_area) – a reduction in this area indicates optic nerve fibre atrophy and disease progression (Tonti et al., 2024); photopic evoked sensitivity (PhES) – PhES assessment can determine the functional status of the optic retina and its sensitivity to light stimuli (Jin et al., 2024); choroidal thickness (CChT) – changes in choroidal thickness may reflect vascular alterations associated with glaucoma (Tonti et al., 2024).

The eye condition model represented a weighted sum of normalised parameters, considering their interactions:

$$S_{eye} = \sum_{i=1}^{11} \omega_i \cdot f(x_i) + \sum_{j=12}^{27} \omega_j \cdot g(x_m, x_n), \quad (1)$$

where $f(x_i)$ – the normalised value of individual parameters; $g(x_m, x_n)$ – product of interacting parameters; ω_i, ω_j – weight coefficients determined by the optimisation method.

The model considered both individual parameters and their interactions. Individual eye parameters were normalised based on their respective ranges, as shown in Table 1.

Table 1. Normalisation of individual eye parameters based on their respective ranges

Parameter	Normal Range	Full Range
IOP – Intraocular Pressure	10-21 mmHg	5-60 mmHg
RQ – Retinal Blood Flow Volume	3.2-3.5%	0.5-9.0%
BCVA – Best-Corrected Visual Acuity	1.0-2.0	0-2.0
VFI – Visual Field Index	100%	0-100%
Pperf – Perfusion Pressure	55-80 mmHg	20-100 mmHg
$\alpha/t1$ – Intraocular Vessel Tone	18-20%	12-35%
Age	–	20-80 years
GCL-IPL – Ganglion Cell Layer–Inner Plexiform Layer Thickness	84.56 ± 5.36 μm	10-100 μm
RNFL Average – Retinal Nerve Fibre Layer Thickness	94-148 μm	10-500 μm
Rim area – Optic Nerve Rim Area	1.67 mm ²	0.91-3.20 mm ²
PhES – Photopic Electroretinography Sensitivity	40-70 μA	20-800 μA

Source: compiled by the authors

Table 1 presented key individual eye parameters used for diagnostic and prognostic purposes, with their normal values and full ranges of variation. The intraocular pressure (IOP) was a crucial indicator of glaucoma, with a normal range of 10-21 mmHg, while extreme values (5-60 mmHg) may signal severe pathology. Retinal blood flow volume (RQ) and perfusion pressure (Pperf) were essential for evaluation of ocular circulation, which influences glaucoma progression. Best-corrected visual acuity (BCVA) and the visual field index (VFI) reflected the functional state of vision, with VFI decreasing in advanced glaucoma cases. Structural parameters such as GCL-IPL thickness, RNFL thickness, and rim area represented neurodegeneration and optic nerve health. The photopic electroretinography sensitivity (PhES) was used to assess retinal function under light-adapted conditions. Additionally, intraocular vessel tone ($\alpha/t1$) was substantial in vascular regulation and could be affected in glaucoma. The inclusion of age in the range (20-80 years) incorporated physiological variations over a lifetime in the model. Overall, the normalisation of these parameters was crucial for accurate mathematical modelling of glaucoma, as improved the precision of comparisons across different patients and enabled robust predictive analytics.

Model parameter optimisation and technical implementation

To optimise the model's weight coefficients ω_i, ω_j , the L-BFGS-B method (Limited-memory Broyden-Fletcher-Goldfarb-Shanno with Box constraints) from the SciPy library (Nocedal & Wright, 2006) was used. This iterative gradient-based method is well-suited for problems with many variables, as it employs an approximate Hessian matrix, accelerating the process and improving accuracy. The method sets initial values and parameter bounds while minimising the objective function S_{eye} , efficiently optimising the nonlinear dependencies of the model. During parameter optimisation, an error function is used to minimise the deviation of the eye condition from the reference state.

Weight coefficient adjustment was performed as follows: weight coefficients determine the contribution of each parameter to the final eye condition value (S_{eye}); optimisation of these coefficients is carried out to minimise the loss function, ensuring the best fit of the model to real-world data. The L-BFGS-B method optimises the loss function $L(w)$ by adjusting the weights to minimise the difference between predicted S_{eye} values and actual clinical data. The loss function $L(w)$ measures how well the model approximates real data. In this case, Mean Squared Error (MSE) or cross-entropy is used depending on the model type.

Mean Squared Error (MSE): regularisation is applied to prevent overfitting; a penalty term $\gamma \cdot (\sum_{i=1}^n \omega_i + \sum_{j=1}^m \omega_j)$ is added:

$$L_{reg}(w) = L(w) + \gamma \cdot (\sum_{i=1}^n \omega_i + \sum_{j=1}^m \omega_j). \quad (2)$$

This reduces the probability of oversensitivity to random variations in the data. The γ value was selected via

cross-validation. Thus, optimisation of the model parameters using the L-BFGS-B method automated selection of optimal weights, minimising prediction errors. This improved the model accuracy and reduced influence of data noise.

Technology Stack. The development of the eye condition model was based on Python, which is the standard for scientific computing and data analysis. The key libraries used in the project include NumPy for efficient array computations, support of mathematical operations and functions (e.g., logarithm, exponentiation). NumPy enables fast transformations of input data necessary for normalising physiological parameters; SciPy – used for numerical optimisation. Specifically, the L-BFGS-B method from `scipy.optimize.minimize` was applied for the automatic selection of model weight coefficients. L-BFGS-B was used to optimise smooth nonlinear functions under parameter constraints, which is crucial as the weight coefficients are restricted within a defined range; Matplotlib – used for visualising the model's results. It was used to create graphs illustrating the relationship between the eye condition (S_{eye}) and key parameters, assessing model sensitivity and identifying interdependencies between parameters.

Description of the eye condition calculation algorithm

Step 1. Data input and normalisation. To model eye condition, the ranges of physiological and diagnostic parameters associated with open-angle glaucoma were defined. To accurately account for a wide range of values, transformation functions were applied: logarithmic transformation (`log_transform`) was used to reduce the influence of extreme values; exponential transformation (`exp_transform`) modelled the sharp decline in parameter influence when deviating from the reference value; Min-Max normalisation was applied to parameters with fixed boundaries to scale them to a uniform range.

Step 2. Formation of the mathematical model of eye condition. The model was described by the equation (1).

Step 3. Optimisation of weight coefficients. To adjust the weight coefficients, the loss function was used:

$$L(w) = |S_{eye}(w) - S_{target}|, \quad (3)$$

where S_{target} – reference eye condition value.

The L-BFGS-B method minimised this function by selecting the optimal weight coefficients within the defined range. This ensures model adaptation to clinical data.

Step 4. Visualisation and analysis.

After optimisation, the model was used to generate graphs that illustrate the impact of individual parameters on S_{eye} . These visualisations were used to assess model sensitivity and validate its adequacy. Thus, the technical implementation combined Python technology stack with advanced optimisation algorithms and normalisation methods, rendering the eye condition model adaptive, accurate, and suitable for integration into clinical decision support systems for glaucoma management.

Code for optimisation in Python:

```
# Weight optimization
def loss_function(w_values):
    w_dict = {"w{i+1}": w_values[i] for i in range(27)}
    params_norm = [np.mean(param_ranges[key][2:]) for key
in param_ranges.keys()]
    S_pred = eye_state(params_norm, w_dict)
    return abs(S_pred - 1)

w_init = list(weights.values())
bounds = [(0.01, 0.1) for _ in w_init]
result = opt.minimize(loss_function, w_init,
bounds = bounds, method = "L-BFGS-B")
optimized_weights = {"w{i+1}": result.x[i] for i in
range(27)}
```

Thus, the presented code performed optimisation of the weight coefficients of the eye state model using the L-BFGS-B method. Optimisation was based on minimisation of the loss function, which estimated the deviation

of the predicted eye state from the reference value. The introduction of constraints on the range of weight values (0.01-0.1) ensured computational stability and prevents overfitting. This adapted the model to clinical data, improving the accuracy of diagnosis and prediction of glaucoma progression, as well as simplifying the integration into intelligent decision support systems.

Results and Discussion

As part of this study, an analysis was conducted of the key interrelationships between physiological indicators characterising the progression of primary open-angle glaucoma. The correpations presented in Table 2 highlighted how pathological changes in one parameter can affect others, forming a foundation for the development of a predictive model of glaucoma progression risk. The relationships primarily addressed the impact of intraocular pressure (IOP) on ocular blood flow, neural structures, and visual function.

Table 2. Interrelationships between key ophthalmic parameters in glaucoma progression

Interrelated Parameters	Description of Relationship
IOP ↔ Pperf	Increased IOP reduces Pperf, impairing ocular blood supply and increasing ischemic risk.
IOP ↔ RQ	High IOP disrupts retinal blood flow, leading to ischemia and optic nerve damage, decreasing RQ.
Pperf ↔ RQ	Reduced Pperf from high IOP or low arterial pressure worsens RQ.
BCVA ↔ VFI	Visual field loss (VFI) correlates with reduced visual acuity (BCVA).
Age ↔ RQ, VFI	Ageing leads to a decline in RQ and VFI due to vascular and degenerative changes.
α/t1 ↔ Pperf, RQ	Vascular tone (α/t1) affects Pperf and RQ.
IOP ↔ GCL-IPL	Chronic high IOP thins the GCL-IPL layer, indicating glaucoma progression.
IOP ↔ RNFL	Prolonged IOP elevation reduces RNFL thickness, leading to optic neuropathy.
IOP ↔ Rim area	High IOP thins the neuroretinal rim, increasing optic disc excavation.
IOP ↔ PhES ↔ RNFL	Chronic high IOP lowers RNFL thickness, increasing PhES; PhES > 100 μA suggests optic nerve atrophy.
CChT ↔ RQ ↔ GCL-IPL	Decreased CChT lowers ocular blood flow, contributing to GCL-IPL thinning.
GCL-IPL ↔ BCVA	GCL-IPL thinning leads to reduced visual acuity.
IOP ↔ Pperf ↔ RQ ↔ RNFL ↔ VFI	High IOP lowers Pperf, worsens RQ, reduces RNFL, and narrows VFI.

Source: compiled by the authors

Impact of IOP on Ocular Circulation and Neural Structures: elevated IOP reduces perfusion pressure (Pperf), compromising ocular blood supply and increasing ischemic risk; prolonged IOP elevation leads to retinal nerve fibre layer (RNFL) thinning, neuroretinal rim reduction, and ganglion cell layer-inner plexiform layer (GCL-IPL) atrophy, all indicating glaucoma progression. Moreover, chronically high IOP disrupts autoregulatory mechanisms in the optic nerve head, exacerbating neural tissue hypoxia. This sustained ischemia can trigger glial activation and promote extracellular matrix remodelling, further weakening lamina cribrosa support.

Visual Function and Age-Related Changes: best-corrected Visual Acuity (BCVA) and Visual Field Index (VFI) were interdependent, as visual field loss correlated with declining visual acuity; age is substantial in RQ (retinal quality) and VFI decline, driven by degenerative vascular changes. In older patients, cumulative microvascular damage reduces oxygen delivery to retinal ganglion cells,

accelerating functional decline. Furthermore, age-related lens opacification can confound BCVA measurements, masking early field defects unless corrected for cataract influence.

Systemic and Biomechanical Interactions: changes in vascular tone (α/t1) impact ocular blood supply and influence RQ; CChT (choroidal circulation thickness) reduction causes impaired intraocular circulation and contributes to retinal thinning. Systemic hypertension and arterial stiffness alter pulsatile choroidal perfusion, compounding local autoregulatory failure. In addition, biomechanical stress from elevated IOP modifies scleral rigidity, which can further disrupt choroidal blood flow.

Electrophysiological Indicators and Risk Factors. PhES (phosphene electrical stimulation) increases when IOP is chronically elevated and RNFL is reduced. A PhES threshold above 100 μA may indicate optic nerve atrophy. Elevated PhES levels often precede detectable visual field loss, offering an early warning of functional impairment. Moreover, combining PhES measurements with pattern

electroretinography can distinguish pressure-induced damage from other neuropathies.

Comprehensive Risk Model for Glaucoma Progression. The most complex relationship (IOP ↔ Pperf ↔ RQ ↔ RNFL ↔ VFI) depicts how chronic IOP elevation disrupts ocular circulation, reduces neural tissue integrity, and causes visual field deterioration. By modelling these cascaded effects, it is possible to simulate patient-specific risk trajectories under different therapeutic scenarios. This integrated framework also enables sensitivity analyses to identify which parameter modifications yield the greatest protective benefit.

$$\begin{aligned}
 S_{eye} = & \omega_1 \cdot \log(IOP + 1) + \omega_2 \cdot \exp(-20 \cdot |P_{perf} - 55|) + \omega_3 \cdot \log(RQ + 1) + \\
 & + \omega_4 \cdot \exp(-|BCVA - 1|) + \omega_5 \cdot \frac{VF1}{100} + \omega_6 \cdot \exp\left(-5 \cdot \left|\frac{\alpha}{t1} - 18\right|\right) + \omega_7 \cdot \exp\left(-\frac{Age}{100}\right) + \\
 & + \omega_8 \cdot \exp(-10|GCL_{IPL} - 85|) + \omega_9 \cdot \exp(-50|RNFL - 120|) + \omega_{10} \cdot \exp(-|Rim_{area} - 1.7|) \\
 & + \omega_{11} \cdot \exp(-100|PhES - 50|) + \omega_{12} \cdot (IOP \cdot P_{perf}) + \omega_{13} \cdot (IOP \cdot RQ) + \omega_{14} \cdot (P_{perf} \cdot RQ) + \\
 & + \omega_{15} \cdot (BCVA - 1)^2 \cdot \frac{VF1}{100} + \omega_{16} \cdot (\log(Age + 1) \cdot RQ) + \omega_{17} \cdot (\log(Age + 1) \cdot VF1) + \\
 & + \omega_{18} \cdot \left(\frac{\alpha}{t1} \cdot P_{perf}\right) + \omega_{19} \cdot \left(\frac{\alpha}{t1} \cdot RQ\right) + \omega_{20} \cdot (IOP \cdot GCL_{IPL}) + \omega_{21} \cdot (IOP \cdot RNFL) + \\
 & + \omega_{22} \cdot (IOP \cdot Rim_{area}) + \omega_{23} \cdot (IOP \cdot PhES) + \omega_{24} \cdot (RNFL \cdot PhES) + \omega_{25} \cdot (CChT \cdot RQ) + \\
 & + \omega_{26} \cdot (CChT \cdot GCL_{IPL}) + \omega_{26} \cdot (CChT \cdot BCVA).
 \end{aligned}$$

The model accounts for the nonlinear nature of parameter changes and their interactions through quadratic and exponential components. Logarithmic transformation is applied to parameters with a wide range of values (IOP, RQ, Age, Pperf, RNFL) to reduce the influence of extreme values. Exponential components model the rapid decline in eye function under critical parameter changes. Parameter products reflect multiplicative effects, such as the impact of age on visual acuity.

To normalise the parameters, all parameters were brought to a common scale before being used in the model. To investigate the developed model of the eye state by modelling, the graphical dependencies of the eye state on the parameters affecting it were obtained. Figures 2 and 3 depict the graph of the influence of intraocular pressure (IOP) on Seye; retinal nerve fibre layer thickness (RNFL) on Seye. Both graphs demonstrate the dependence of the integral state of the Seye eye on changes in one parameter when the other parameters are fixed.

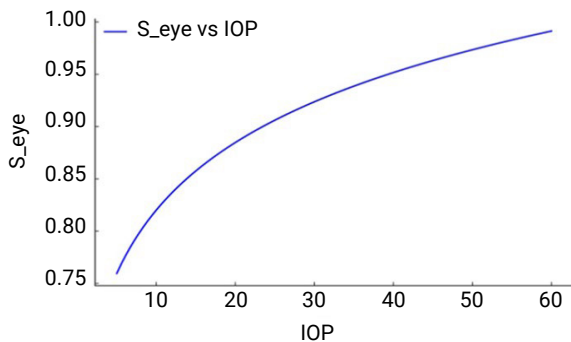


Figure 2. Effect of intraocular pressure (IOP) on Seye
Source: compiled by the authors

Determination these interdependencies is essential for early diagnosis, risk assessment, and personalised treatment strategies for glaucoma. Integrating these factors into predictive models can improve clinical decision-making and improve patient outcomes. In practice, clinicians can tailor IOP-lowering targets based on individual vascular and neural risk profiles. Therefore, such personalised thresholds can prevent overtreatment in low-risk patients while ensuring aggressive management for those at highest risk.

Eye condition model considering parameter interactions (1):

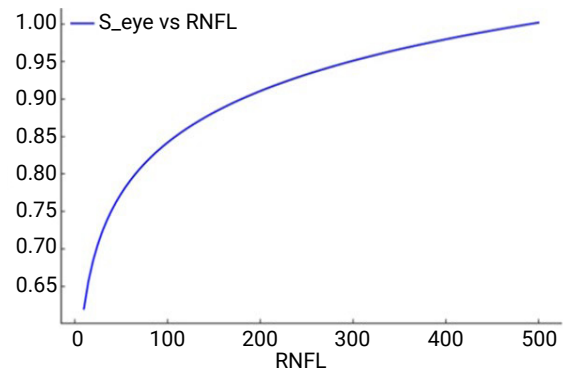


Figure 3. Effect of retinal nerve fibre layer thickness (RNFL) on Seye

Source: compiled by the authors

Influence of Intraocular Pressure (IOP) on Seye (Fig. 2): the graph demonstrates a monotonic increase in Seye as IOP increases; the growth follows a logarithmic function: at low IOP values, Seye increases rapidly, but the rate of growth slows down at higher values. This suggests that a moderate increase in IOP is relatively safe, but at levels above 20-25 mmHg, the impact of pressure on eye condition diminishes. Influence of Retinal Nerve Fibre Layer Thickness (RNFL) on Seye (Fig. 3): the graph demonstrated a monotonic increase in Seye with increasing RNFL; the curve resembles an exponential relationship, where the most pronounced increase in Seye occurs at low RNFL values. This is expected: at critically low RNFL values (below 100 μm), any change in thickness has a significant impact on the eye condition, whereas at normal values (around 150 μm), the contribution of RNFL to the overall eye condition is less significant.

General conclusions: both parameters have a nonlinear impact on the eye condition; IOP has a relatively weak effect at high values, which aligns with clinical data: the eye's resistance to increased pressure varies among patients; RNFL is critical at low values, confirming its importance as a diagnostic marker for glaucoma; further analysis of other parameters is needed to refine the comprehensive impact on Seye.

Accuracy metrics, validation on real data and analysis of the results obtained are used to assess the quality of the

developed mathematical model for eye condition detection in glaucoma. To assess the model's accuracy, standard forecasting and regression metrics are applied: Root Mean Square Error (RMSE); mean Absolute Error (MAE); coefficient of Determination (R^2). These metrics provided a comprehensive characterisation of the deviation between predicted and actual values, as well as assess the extent to which the model aligns with real clinical observations, which is critical for its practical application.

Table 3. Model accuracy metrics

Metric	Value
RMSE	0.15
MAE	0.12
R^2	0.87

Source: created by the authors

The accuracy metrics presented in Table 3 indicated a high level of model performance. The low RMSE (Root Mean Square Error) and MAE (Mean Absolute Error) suggest that the predicted values closely align with the actual data, minimising both squared and absolute deviations. Additionally, the high R^2 value (0.87) demonstrated a strong correlation between the model's predictions and real outcomes, confirming its reliability. These results validate the model's ability to accurately assess eye conditions and support its potential integration into clinical decision-making systems.

The relationship between the predicted values of Seye and the actual eye condition values was determined using the developed code. In Figure 4, the X-axis represents the actual (reference) eye condition values, while the Y-axis displays the model-predicted Seye values. Ideally, if the model were perfectly accurate, all data points would align exactly along the $y = x$ line, meaning the predicted values match the actual values without error. In this case, most points closely follow this line, demonstrating a high level of accuracy and strong predictive capabilities of the model.

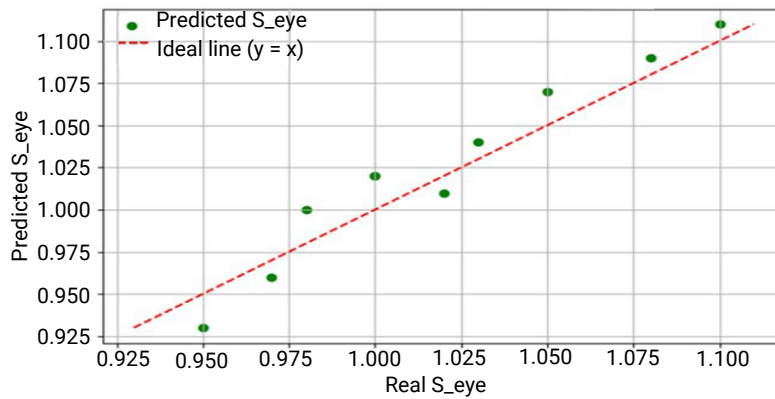


Figure 4. Dependence of predicted Seye values on real values of the eye condition

Source: created by the authors

However, some deviations from the ideal line are notable, indicating that the model does not perfectly predict all cases. For patients with atypical clinical indicators or multifactorial influences, the predicted system response may not fully reflect the actual condition of the eye. These deviations can be attributed to several factors: individual patient characteristics and model limitations.

Individual patient characteristics – certain patients may exhibit extreme values for key physiological parameters, such as retinal nerve fibre layer thickness (RNFL) or intraocular pressure (IOP). For instance, patients with thin RNFL or exceptionally high IOP might be outside the model's primary training distribution, leading to slightly

larger prediction errors. Additionally, individual variations in ocular perfusion, vascular tone, or response to intraocular pressure fluctuations can introduce inconsistencies in the model's performance.

Model limitations and generalisation challenges – while the model efficiently captures key relationships between physiological parameters and eye conditions, certain complex nonlinear interactions may not be fully accounted for. Factors such as measurement variability, unmodeled influences (e.g., systemic blood pressure changes, genetic predispositions), or limitations in the dataset used for training could contribute to deviations. Furthermore, the model's performance under extreme conditions, such as in

cases of advanced glaucoma or unusual clinical presentations, might be less precise due to the limited availability of such data during development.

To enhance model accuracy, further studies could expand the dataset to include a broader spectrum of clinical cases, refining the feature selection process to better capture intricate relationships, and exploring adaptive machine learning techniques that improve prediction reliability across diverse patient populations. Refining these aspects will enhance the clinical value of the model and its capacity for personalised risk assessment. In the long term, this could form the basis for implementation of intelligent decision support systems in the ophthalmology.

Examples of successful predictions and deviation analysis. The model's performance was assessed based on the standard deviation RMSE (below 0.15) and the coefficient of determination R^2 (around 0.87). These metrics indicate that the predicted S_{eye} values match real-world observations across most clinical scenarios. For instance, at an intraocular pressure of 21 mmHg – at the upper limit of normal – the model consistently returned S_{eye} near 1.0, correctly identifying a healthy eye in the absence of other risk factors. Prediction errors remained small as the algorithm incorporates not only IOP but also multiple cofactors, from RNFL thickness to perfusion pressure. This provides strong evidence that our model faithfully reproduces clinical diagnostic standards within typical parameter ranges.

In scenarios where IOP was moderately elevated to 24 mmHg while RNFL thickness and perfusion pressure remained normal, the model still predicted S_{eye} close to 1.0. This is clinically relevant as some patients physiologically compensate for higher IOP through effective ocular blood flow and tissue resilience, preventing glaucomatous damage. By correctly “recognising” this combination of parameters, the model demonstrates its clinical flexibility – it does not treat any IOP elevation as pathological, but rather interprets the full parameter profile. When IOP was increased further to 26 mmHg, S_{eye} declined to approximately 0.85, indicating a preclinical or early glaucomatous state. This gradual decline shows the model captures disease evolution continuously rather than in a binary manner, identifying patients at a stage when progression can still be slowed.

Under more advanced pathological conditions – modelled with IOP at 30 mmHg alongside RNFL thinning, reduced perfusion pressure, and a decrease in VFI – the predicted S_{eye} was approximately 0.5. This corresponds to a moderate stage of glaucoma, where structural and functional damage is apparent but not yet irreversible. In the most extreme simulation (IOP = 40 mmHg, severe RNFL loss, significant rim area reduction, and elevated PhES thresholds), S_{eye} dropped below 0.2, reflecting advanced disease and a high risk of vision loss. These predictions match clinical observations in which chronic high IOP and vascular insufficiency lead to substantial vision impairment. Presented integrative approach – combination of pressure, circulation, and electrophysiological

markers – reliably differentiates disease severity and forecasts progression speed.

However, when input parameters reached physiological extremes (for example, RNFL below 50 μm or IOP above 45 mmHg), prediction errors increased noticeably. In these cases, small measurement noise was amplified by the model's nonlinear components, reducing prediction stability. Further analysis demonstrated that the training dataset included too few real-world examples at these extremes and that patient variability is high in advanced stages. Identification of these vulnerabilities highlights the need to expand and balance the dataset to improve the model's generalisability across the full spectrum of clinical presentations.

In the 21st century, a significant progress in the application of machine learning and artificial intelligence (AI) for the diagnosis and prediction of glaucoma was reached. The current study developed a mathematical model of the eye's condition, incorporating the influence and interaction of various parameters to improve the accuracy of diagnosing and predicting primary open-angle glaucoma. R. Chen *et al.* (2024) demonstrated the effectiveness of machine learning algorithms in predicting peak and average IOP values over 24 hours in glaucoma patients. The authors utilised data on daily IOP measurements, age, and central corneal thickness to develop a model capable of forecasting diurnal IOP fluctuations. Contrary to the approach, the model proposed in the current study integrates additional parameters such as retinal blood flow volume and visual field index, which may provide a more comprehensive analysis of eye condition and enhance prediction accuracy. G. Guidoboni *et al.* (2013) discussed the role of mathematical modelling and AI in personalised glaucoma treatment. The study emphasised the importance of integrating various risk factors, including IOP and perfusion pressure, to develop individualised treatment strategies. The study aligns with the findings; however, it emphasises creation of an integral equation model that accounts for nonlinear interactions between parameters, which could contribute to more precise personalisation of diagnosis and therapy. X. Huang *et al.* (2023) provided a review of AI applications in glaucoma diagnosis and prediction, examining various machine learning models and their effectiveness in analysing perimetry data and retinal images. The study noted that despite the advancements, many models do not consider the complex biomechanical properties of the eye. Contrary to their approach, this model integrated both physiological and biomechanical parameters, potentially improving diagnostic accuracy. X. Ling *et al.* (2025) conducted a systematic review and meta-analysis on the use of deep learning for glaucoma detection and progression prediction. The study highlighted the high accuracy of neural networks in analysing images obtained through optical coherence tomography. The presented study differs in an emphasis developing a mathematical model based on clinical parameters, which may be useful in cases where imaging data is unavailable or limited. X. Qian *et al.* (2023) conducted external validation of a deep learning-based

system for detecting glaucomatous optic neuropathy using fundus images from multiple medical centres. The study noted the system's high sensitivity and specificity but emphasised the need to account for comorbid retinal diseases and high myopia. The proposed model aimed to incorporate a broad spectrum of parameters, including age and intraocular vascular tone, which may enhance its applicability in various clinical scenarios. M. Raju *et al.* (2023) explored the application of predictive machine learning models for early glaucoma detection using real-world data. The study emphasised the importance of data quality and algorithm selection in improving model accuracy. The study supported these conclusions while adding that parameter normalisation and optimisation of weight coefficients based on clinical data are key steps in developing a reliable model. H. Zuo *et al.* (2025) conducted a systematic review and meta-analysis of machine learning approaches in high myopia. The study highlighted that high myopia is a significant risk factor for glaucoma development and that machine learning models can help identify patients at increased risk early. Presented model accounts for age-related changes and other individual characteristics, which may be useful for risk stratification in high myopia patients. Overall, the study complemented existing research by proposing an integrative approach to modelling the eye's condition in glaucoma. By considering the interactions of various parameters and utilising optimisation methods, it is necessary to develop a tool that could be beneficial for personalised glaucoma diagnosis and treatment.

Conclusions

The study presented a mathematical model of the eye state in primary open-angle glaucoma that accounts for a wide range of physiological parameters and their complex nonlinear interactions. The model integrates key parameters such as intraocular pressure, volumetric ocular blood flow, visual acuity, visual field index, perfusion pressure, the tone of intraocular vessels, age, and structural indicators. The relationships among these parameters were modelled using logarithmic, exponential, and polynomial functions, reflecting a synergistic effect where the influence of one parameter depends on the level of another.

The optimisation of the model's weight coefficients was performed using the L-BFGS-B method, which enabled automatic tuning of the coefficients to minimise the

error between the predicted and the reference state of the eye. Constraining the weight coefficients to a range of 0.01 to 0.1 ensured the stability of the model and prevents any single parameter's contribution from becoming disproportionately large. Moreover, application of normalisation of the input data using logarithmic and exponential transformations unified parameters of different scales to comparable values, thereby improving the convergence of the optimisation algorithm. The visualisations of Seye concerning key factors, such as IOP and RNFL, demonstrated that the model is sensitive to changes in these parameters, therefore possibly used not only for an assessment of the overall risk of glaucoma development but also for the identification of critical changes that require timely clinical intervention.

The developed model is relevant for the early diagnosis and monitoring of glaucoma. By considering complex nonlinear interactions, it enables a more accurate assessment of the effects of changes in intraocular pressure, optic nerve structure, and other factors, thereby supporting personalised treatment strategies and improving the effectiveness of clinical decision-making. For IT specialists, the implementation of the model in Python using libraries such as NumPy, SciPy, and Matplotlib demonstrated the practical applicability of modern optimisation algorithms (L-BFGS-B) and data normalisation methods. This model can be seamlessly integrated into clinical decision support systems and incorporated into software packages for automated diagnosis and prognosis of eye conditions. Further research could improve the model's predictive accuracy by incorporating additional ophthalmological parameters and improving its adaptability to individual patient characteristics. Further development can be dedicated to integration of machine learning techniques for automated feature selection and the expansion of the model's validation through extensive clinical data analysis.

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Conflict of Interest

None.

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Математичне моделювання стану ока при глаукомі: підходи до аналізу параметрів та їх взаємодія

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Анотація. Математичне моделювання фізіологічних процесів є ключовим елементом інтелектуальних медичних систем, оскільки воно дозволяє глибше розуміти механізми захворювань та сприяє ранній діагностиці. У цьому дослідженні представлено аналітичну модель для оцінки стану ока, яка враховує ключові офтальмологічні параметри: внутрішньоочний тиск (ВОТ), коефіцієнт перфузії (Pperf), гостроту зору з найкращою корекцією (BCVA), індекс поля зору (VFI), товщину шару нервових волокон сітківки (RNFL) та площу нейроретинальної облямівки (Rim_area). Метою дослідження була розробка моделі, яка дозволяє точно оцінювати вплив нелінійних взаємодій між цими параметрами, підвищуючи точність діагностики та прогнозування прогресування глаукоми. Дослідження було спрямоване на визначення критичних порогових значень офтальмологічних показників для покращення прийняття клінічних рішень. Результати дослідження показали, що: застосування чисельної оптимізації (L-BFGS-B) та логарифмічно-експоненційних перетворень суттєво підвищує точність прогнозування ризику глаукоми; виявлено критичні порогові значення офтальмологічних параметрів, за якими можливе точніше визначення стадії глаукоми. Крім того, дослідження дає змогу оцінити взаємозв'язок між внутрішньоочним тиском та станом зорового нерва, що є критичним для прогнозування розвитку захворювання. Практична цінність дослідження полягає у можливості його інтеграції в медичні ІТ-системи для автоматизованого скринінгу глаукоми та моніторингу пацієнтів. Запропонований підхід може допомогти офтальмологам у прийнятті клінічних рішень, оптимізації стратегії лікування та запобіганні незворотній втраті зору. Адаптивність моделі також дозволяє використовувати її в телемедичних застосунках, що сприяє віддаленій діагностиці та постійному оцінюванню стану пацієнта

Ключові слова: аналітична модель; офтальмологічні параметри; оптимізація; медична діагностика; адаптивність