**Journal homepage:** https://itce.vn.ua/en

**Information Technologies and Computer Engineering** **Vol. 23, No. 2. 2026**

**Article's History:** Received: 26.01.2026 Revised: 25.05.2026
Accepted: 25.06.2026 Published: 03.08.2026

ISSN 1999-9941; e-ISSN 2078-6387
UDC 004.5
DOI: 10.31649/vitce/2.2026.67

# Adaptation of Fitts' law in three-dimensional interfaces

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**Abstract.** The lack of a consensus choice of the calculation model for adapting Fitts' law in the development of three-dimensional interfaces, and the heterogeneity of the research methodology on the relevant topic, makes it difficult to compare their results. The purpose of the study was to determine the most effective computational model of user experience in the development of spatial interfaces in virtual and reality in two-dimensional and three-dimensional environments. For this purpose, a meta-analysis of a representative sample of scientific papers was carried out, distributed according to measurement criteria (one-dimensional, two-dimensional, three-dimensional) and research areas, and ordered by the percentage of citations. According to the results of formalised distribution criteria, the Shannon equation proposed by Mackenzie (77.8%) and the Machuka angular equation (28.7%) had the highest citation rate in three-dimensional works. The original Fitts equation in this context, even as a primary source, had 4% relevance, and the most effective solution in a two-dimensional plane under Mackenzie's authorship – only 2%. It was revealed that the most relevant are three-dimensional models with the involvement of additional parameters, including trigonometric angular coefficients and sets of coordinates for determining the depth or complex trajectories of achieving goals, which was confirmed by high citation rates of the corresponding approaches. It was established that the effectiveness of each of the demonstrated adaptations of Fitts' law was not determined only by the type and number of selected parameters, since no less important role is played by the process of comparing theoretical calculations with empirical data obtained as a result of user interaction with hardware complexes, the dynamics of availability of which for a wide range of researchers radically changed during a short period of time (10-20 years), considering the general chronological range of the considered papers (1954-2025). The practical significance of the research lies in the fact that the results obtained can be used to develop user interfaces in virtual and augmented reality tools, form standards, and methodological recommendations when working with spatial interfaces, and improve feedback when analysing user experience

**Keywords:** spatial interfaces; virtual reality; augmented reality; user interface; sensory-motor behaviour

## Introduction

Fitts' law is an integral sensory-motor principle in the construction of effective human-machine interaction interfaces. The application of this law for working with two-dimensional interfaces proved its effectiveness and developed a theoretical basis for the principles of ergonomic design, methods for evaluating user behaviour, and also revealed the most effective means of entering and reading commands. Along with the emergence and rapid development of spatial interaction technologies, in particular virtual reality (VR) and augmented reality (AR) technologies, the need

**Suggested Citation:**
Karmanov, R., & Zubretska, N. (2026). Adaptation of Fitts' law in three-dimensional interfaces. *Information Technologies and Computer Engineering*, 23(2), 67-77. doi: 10.31649/vitce/2.2026.67.

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to expand and adapt the consensus theoretical methodology has become more relevant. Standardisation of methods for using Fitts' law in virtual and augmented environments is an essential condition for developing new-generation interfaces, optimising work with three-dimensional digital systems, and ensuring safe interaction with them.

However, there are a number of differences between three-dimensional environments and their two-dimensional predecessors, which create additional factors that affect both the accuracy and execution time of movements, and the interface architecture as a whole. The key factors are the depth of space, the dynamics of viewing angles, the smoothness of the display, and the variety of controls: spatial scanning of objects using appropriate sensors, neural network modelling or beam pointers of choice. As a result, the original form of Fitts' law, which was based only on distance indicators and the proper size of an object on a plane, requires rethinking the concept and updating it. Several attempts were made to modify the model, including the use of angular variables, three-dimensional parameters, calculation of the trajectories of rotational and translational movements, which, however, did not allow forming a consensus approach in calculating the efficiency of object movement in a virtual environment.

S. Okuuchi *et al*. (2023) and A. Esteves *et al*. (2020) presented a number of versions of the adaptation of Fitts' law in three-dimensional environments, where the main means of testing are VR or AR systems. Depending on the conditions and specifics of the experiments, the researchers offer different versions of the equations, introducing additional criteria, in particular, the depth of objects, features of the spatial orientation of users, and the size of the finger and other means of entering information (Minakata *et al*., 2019; Gabel *et al*., 2023; Wagner *et al*., 2023). Given the significant differences in the direction, means, and years of research, some models are partially or completely incompatible in their mutual use. The difference in deviations in results is significant in many scenarios, which was confirmed in studies by D. Wolf *et al*. (2018), D. Yu *et al*. (2019) and L. Zhao *et al*. (2020). This is conditioned by the fact that at the moment there is no generally accepted methodology for conducting such experiments, mainly due to the relative originality of the scientific area, which is supported by M.D. Barrera Machuca & W. Stuerzlinger (2019) and A.A. Ullah *et al*. (2023).

A generalised unsolved scientific problem is the lack of a universal three-dimensional adaptation of Fitts' law, which accurately describes and predicts the ergonomics of human interaction with the spatial interface, covering as many individual motor characteristics of users as possible. This creates the need to adapt the works of Paul Fitts to the conditions of working with spatial computers. Therefore, the purpose of the study was to investigate the available publications on adaptations of Fitts' law for two-dimensional and three-dimensional planes, and identify the most effective ones for each level of spatial interaction. This allowed eliminating methodological heterogeneity, systematising approaches to the development of spatial user interfaces, choosing the most effective one and confirmed by test results. The main hypothesis of the study was that the analysis of all current adaptations of Fitts' law in three-dimensional space to identify the most effective model will create a theoretical basis for the development of structural, convenient, and secure spatial user interfaces, accelerating the development of both software and hardware solutions in the field of spatial computers.

To achieve this goal, the following tasks were set:

- to search for publications that will include variations of adaptations of Fitts' law based on open sources;
- to conduct a statistical analysis of the citation of the authors of three-dimensional adaptations of Fitts' law and determine the most effective one by the percentage of citation.

The research was conducted based on scientific databases, without creating or testing new software or hardware solutions. The citation index was chosen as a marker of the correctness of the author's hypothesis. The course of the study was divided into three main stages:

- search for scientific and technical literature: an initial review of the results of queries to open scientific databases (119 units): ACM Digital Library (40 units), IEEE Xplore Digital Library (25 units), SpringerLink (36 units), ScienceDirect (18 units) to accumulate all relevant material, and the initial exclusion of papers;
- main stage of material selection: all found publications that successfully passed the initial search and selection stage were analysed for the availability of the material text through direct links, or by searching by available identifiers, authors or fragments (50 units). Publications whose text was not available for viewing, or whose context did not correspond to the purpose of the study, were excluded from the set and did not pass to the next stage (69 units);
- final review and ranking of authors and material: publications that meet the main selection criteria were considered individually for general systematisation of adaptation models and their indexing by citation percentage to identify the most used ones. Several authors could work on the same equation, or one author could consider several equations, which justified the expediency of displaying each publication separately.

## Unresolved research issues of contemporary scientific discourse

Fitts' law is one of the main models for predicting user motor behaviour when interacting with computers via a graphical interface (Fitts, 1954). In contemporary development practice, the patterns identified by the researcher allow building an interface architecture for devices based on a two-dimensional display even without preliminary testing, forming an ergonomically optimised user experience, regardless of the type of input tools. Along with the advent of three-dimensional types of interfaces, there was a need to rethink generally accepted methodological standards, which differ in the chosen accents and philosophies depending on the nature of experiments and the authors' own vision.

I.S. MacKenzie (1992), S. Zhai (2004), R.W. Soukoreff & I.S. MacKenzie (2004) directed efforts to generalise Fitts' characteristic conclusions for two-dimensional space, relying on factors such as the vector of movement, the shape and size of the target, and the distance to it. The results showed that the primary criteria of *ID* did not fully determine the real time of the manoeuvre, which was most representative in cases with asymmetric shapes and objects of complex shapes (Welford, 1960; Hoffmann, 1995; Cha & Myung, 2013). Despite the fact that these studies were conducted mainly in laboratory conditions, most of the results are an average sample of user tests, which may not fully cover specific cases of individual motor features, some of which were currently considered integral in the development of interfaces (MacKenzie, 1989).

V. Schwind *et al*. (2019), J. Sehrt *et al*. (2023), N.T. Tanacar *et al*. (2023) considered individual adaptations that considered the difficulty and variability of user movement paths in three-dimensional space. The researchers insist that the linear dependence of the *ID* on Movement Time (*MT*) inherent in two-dimensional conditions is significantly more complex for different user groups in a three-dimensional environment (MacKenzie, 1992; Murata & Iwase, 2001; Barrera Machuca & Stuerzlinger, 2019). At the same time, the problem of unifying the calculation model remains unresolved, and some of the current basic user features are not considered in calculations (Welford, 1968; Hoffmann, 1995; Accot & Zhai, 2003).

Special attention should be paid to the group of developments by D.E. Meyer *et al*. (1988), S. MacKenzie & W. Buxton (1992) and L.D. Clark *et al*. (2020), united by the research topics of personalised three-dimensional adaptations considering the variability of user motor behaviour. These materials serve as an important example of considering the accuracy of personalisations in modelling motor problems, although these approaches are situational and do not cover more usable scenarios than their generalised counterparts (Watson *et al*., 2003; Shoemaker *et al*., 2012; Janzen *et al*., 2016). In addition, in most cases, there is no systematic analysis of individual parameters for generalised equations (Li *et al*., 2020; Xiao *et al*., 2024; Amini *et al*., 2025).

Thus, the following issues remain unresolved:

- lack of a universal generally accepted calculation model demonstrating equivalent or approximately equal efficiency in both two-dimensional and three-dimensional problems;
- the results of experiments of different authorship are difficult to compare due to the heterogeneity of methods, the difference in chronology, and, accordingly, the hardware capabilities for implementing testing;
- most tests are based on simplified and averaged user behaviour, without considering the dynamics of perception and individual characteristics.

The reasons for not solving the above-mentioned problems should be divided into objective and subjective. Objective reasons include the difficulty of formalising motor behaviour while covering as many separate scenarios and individual characteristics as possible, and a high difference in the development of applied tools at the time of testing. Subjective reasons include the author's preference for traditional approaches and a shift in focus to verifying outdated models, neglecting their rethinking and updating.

**Fitts' law and its subsequent reinterpretations**

Fitts' 1954 research was based on a combination of information theory and the human motor system. It was based on information theory that he formulated the primary law that determines the difficulty of motor tasks in the VMC process. *ID* of a motor task was determined by the equation (1):

$$ID = -log_2\left(\frac{W}{2A}\right) = log_2\left(\frac{2A}{W}\right), \tag{1}$$

where *W* – width to the goal of the motor task, *A* – target size. The bits serve as the unit of measurement for ID. The essence of the equation is as follows: the difficulty of achieving a goal is directly proportional to the distance to it, and inversely proportional to its size. In addition, Fitts developed the equation (2) for determining the Index of Performance (*IP*) for a motor task completed within time t (seconds):

$$IP = -\frac{1}{t}log_2\left(\frac{W}{2A}\right). \tag{2}$$

*IP* is measured in bits/s. In more recent studies, it has also been suggested to name *IP* bandwidth (Throughput, *TP*) (Zhai, 2004; Soukoreff & MacKenzie, 2004). According to hypothesis by P.M. Fitts (1954), the *IP* of a motor task remains constant for a wide range of values *A* and *W*. According to the results of his experiments involving three different motor tasks, the values of *IP* were not absolutely equivalent, but the deviations among the results were insignificant. Substituting equation (1) into equation (2), the equation (3) is obtained:

$$t = \left(\frac{ID}{IP}\right). \tag{3}$$

The equation demonstrates a direct proportional dependence of time and difficulty of *ID*. It follows that in order to effectively apply Fitts' law, it is necessary to find ways to reduce the value of *ID*. The simplest solution is to increase the target itself, and position the object as close to the manipulator as possible. P.M. Fitts & J.R. Peterson (1964) proposed the equation (4) for predicting the *MT* required to perform a sports task based on the *ID*:

$$MT = a + b \cdot ID, \tag{4}$$

where, *a* – point of intersection with the axis, *b* – its slope, which varies depending on the specific application. It is this equation that underlies Fitts' law, demonstrating a linear correlation of *MT* with the complexity of the task, which has become a fundamental principle of interaction architecture in interface development.

In the course of further research, attempts were made to improve and rethink Fitts' law, focusing on current requirements and challenges (MacKenzie, 1992; Hoffmann, 1995; Cha & Myung, 2013). First of all, the movement vectors from the starting point to the target were limited to the two-dimensional plane, as a result of which the applicability of the original Fitts model was limited only to simple movements. This is conditioned by the fact that in real-world scenarios, translational movement on the plane is less common than rotational and other forms, which requires consideration of three-dimensional models to solve this kind of complexity. Due to the variety of variables and scenarios in three-dimensional movement, it is extremely difficult to generalise motor behaviour using a single simulation model.

**Fitts' law in contemporary studies**

The available forms of three-dimensional adaptation of Fitts' law depend on the specific conditions in which they manifest themselves effectively. Despite this, Fitts' law continues to serve as the most promising basis for further research. Figure 1 shows a graph of the annual number of Fitts citations in the scientific and technical literature from 2000 to 2023 (Amini *et al.*, 2025).

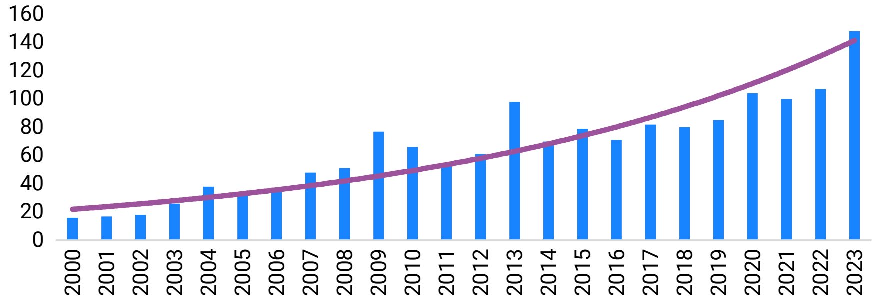


**Figure 1.** Dynamics of the use of Fitts' law in publications (2000-2023)
**Source:** developed by the author based on M. Amini *et al.* (2025)

As can be seen from the graph (Fig. 1) despite the slight difference in the number of citations over some adjacent years, the trend line remains constantly growing, which confirms the constant increase in researchers' interest in Fitts' law, and confirms its relevance and effectiveness as a theoretical basis for the development of modern digital interfaces. In all these studies, Fitts' law was specifically used as part of calculations, and its application covered at least one of the categories systematised in Table 1.

**Table 1.** Main areas of application of Fitts' law in research

| No. | Research area | % of research | Source |
|---|---|---|---|
| 1 | Interaction and evaluation of the system | 62.3 | A. Esteves *et al*. (2020); J. Gabel *et al*. (2023); K. Minakata *et al*. (2019); U. Wagner *et al*. (2023); D. Wolf *et al*. (2018); D. Yu *et al*. (2019); L. Zhao *et al*. (2020). |
| 2 | Other factors affecting the interaction | 26.2 | M.D. Barrera Machuca & W. Stuerzlinger (2019); A.U. Batmaz *et al*. (2022); A.U. Batmaz *et al*. (2023); X. Lv *et al*. (2021); A.A. Ullah *et al*. (2023). |
| 3 | Feedback evaluation | 13.9 | A.U. Batmaz *et al*. (2020); A.U. Batmaz & W. Stuerzlinger (2021); A.U. Batmaz *et al*. (2022); V. Schwind *et al*. (2019); J. Sehrt *et al*. (2023); N.T. Tanacar *et al*. (2023). |

**Source:** developed by the author based on M. Amini *et al.* (2025)

Based on the results of systematisation, it follows that in most studies, Fitts' law was used to evaluate a method or system of interaction. In rare cases, it was also used to evaluate different types of feedback, such as auditory (Batmaz & W. Stuerzlinger, 2021; Batmaz *et al.*, 2022) or tactile communication (Tanacar *et al.*, 2023). There are also isolated studies of the effect of secondary factors on user experience, such as the effect of rotational jitter, L. Kohli *et al*. (2012) the correlation of work productivity and space deformability was established, and M.D. Barrera Machuca & W. Stuerzlinger (2019), who focused on the impact of stereo display disadvantages. All other small and isolated documented cases of Fitts' law were not included in the list for a number of reasons, namely, inconsistency of any of the topics considered, blurring of wording and conclusions, or lack of prospects for application in three-dimensional space.

The list of scenarios for applying Fitts' law was significantly expanded due to the integration of probability theory, due to which researchers were able to update the model and its predictive capabilities in dynamic conditions. Table 2 shows a set of the most conventional extended equations formulated during the development of adaptations of Fitts' law in contemporary research (Xiao *et al.*, 2024).

**Table 2.** Classical study of Fitts' law in two-dimensional domains

| Source | *ID* equation | Measurement |
|---|---|---|
| P.M. Fitts (1954) | $ID = log_2\left(\frac{2A}{W}\right)$ | 1/2D |
| A.T. Welford (1960) | $ID = log_2\left(\frac{A}{W} + 0.5\right)$ | 2D |
| I.S. MacKenzie (1989) | $ID = log_2\left(\frac{A}{W} + 1\right)$ | 2D |
| E.R. Hoffmann (1995) | $ID = log_2\left(\frac{2A}{W+F}\right)$ | 2D |

**Source:** developed by the authors based on H. Xiao *et al.* (2024)

According to open sources, it is A.T. Welford (1960) who was the first to improve the original equation by replacing double distance with a constant of 0.5 for the *ID*. Later, I.S. MacKenzie (1989) chose an alternative approach, questioning the difference between deviations in Fitts' study and equations in information theory. The researcher developed his own equation, and then repeated the construction of a graph of Fitts' results in 1954, proving a higher accuracy of calculations than in the original. The main difference between this approach was replacing the constant value with 1. Unlike A.T. Welford (1960), I.S. MacKenzie (1989) justified his choice by the need for the cursor to reach exactly the centre of the target, since the set of tangent cases contains a significant number of deviations and errors that cannot be considered successful interaction with the object. However, both equations improved on previous results in their respective experiments. The experiment by E.R. Hoffmann (1995) supplemented the equation with a variable user finger width (F), thus demonstrating the importance of this factor in combination with the target width (W) and its distance (A). With this approach, the inverse Fitts problem equation showed improved results. The success of Hoffmann's research allowed adapting this method in three-dimensional space, complementing the equations with the depth of the goal. Among the adaptations considered, the most popular and universal equation in a two-dimensional plane is the Mackenzie equation, since it gives the most effective results in flexible use cases (Holmes, 2016; Hayes, 2016; Okuuchi *et al.*, 2023).

The influence of the object thickness factor in predicting user interaction was the starting point for expanding limited two-dimensional studies to three-dimensional ones. E.R. Hoffmann (1995) provided a sufficient basis for adapting Fitts' law in three-dimensional space. Since the complexity of analysing human movement in three dimensions is much higher, adaptations are necessary for both the difficulty factor and the time of movement equation. Considering the above conditions, a number of researchers in the course of their experiments were able to trace certain patterns and draw appropriate conclusions. Table 3 showed the most significant results of three-dimensional adaptations and their authors.

**Table 3.** Classical equations for adaptations of Fitts' law in three dimensions

| Source | *MT* | *ID* | Measurement |
|---|---|---|---|
| P.M. Fitts (1954) | $MT = a + b \cdot ID$ | $ID = log_2\left(\frac{2A}{W}\right)$ | 1/2D |
| A. Murata & H. Iwase (2001) | $MT = a + b \cdot ID$ | $ID = log_2\left(\frac{d}{s} + 1\right) + c \cdot sin\,\theta$ | 3D |
| Y. Cha & R. Myung (2013) | $MT = a + b\theta_1 + c\,sin\,\theta_2 + d \cdot ID$ | $ID = log_2\left(\frac{2D}{W+F}\right)$ | 3D |
| M.D. Barrera Machuca & W. Stuerzlinger (2019) | $MT = a + b \cdot ID + c \cdot CTD$ | $ID = log_2\left(\frac{A}{W} + 1\right)$ | 3D/VR |

**Source:** developed by the authors based on H. Xiao *et al.* (2024)

In the early stages of research, A. Murata & H. Iwase (2001) formulated their own version of three-dimensional adaptation, which did not bypass the modification *ID*. Based on Mackenzie's equation, the equation was extended using the sinusoidal function of the angle of movement (θ), where θ – value of the angle directed to the vector of movement. An arbitrary constant (c) was applied using linear regression. This allowed Murata's equation to improve Fitts' results in predicting *MT*, albeit with a slight difference. Later, with the participation of Y. Cha & R. Myung (2013) an alternative equation was presented that considers the angle of inclination (α), which refers to the angle between the object and the horizontal plane, and the orientation angle (β), as the value of positioning in the coordinate system, where the user's hand serves as the centre. The main goal was to improve the accuracy of modelling three-dimensional movement by improving the spherical coordinate system, but the results showed that the involvement of the variable Sin(θ) had minimal impact. Subsequently, M.D.B. Machuca & W. Stuerzlinger (2019) used VR equipment for practical experiments to identify unexpected effects from the impact of display inaccuracies, including those related to changes in target depth (CTD). The resulting equation significantly increased the efficiency of predicting the time of movement, becoming an important work for the development of spatial computing.

Fitts' law and its subsequent adaptations remain an unresolved component of three-dimensional research, since the primary equation was developed within the framework and properties of two-dimensional space. With such uncertainty, the choice of calculation equation has a fundamental impact on research results. Despite the variability of equations in experiments, statistical observations show certain patterns in the choice with certain errors in versions among the contingent of researchers. Table 4 shows all the identified options for adapting Fitts' law, and the corresponding percentage of their citations in the papers.

**Table 4.** Citation rating of adaptations of Fitts' law in three-dimensional experiments

| No. | Source | Equations | % of citations |
|---|---|---|---|
| 1 | Shannon (MacKenzie, 1992) | $MT = a + b\, log_2\left(\frac{A}{W} + 1\right)$ | 77.8 |
| 2 | Corner (Kopper *et al.*, 2010; Barrera Machuca, 2023) | $MT = a + b\, log_2\left(\frac{\alpha}{\omega^k} + 1\right)$ | 28.7 |
| 3 | P.M. Fitts (1954) | $MT = a + b\left(\frac{2A}{W}\right)$ | 4.0 |
| 4 | M.D. Barrera Machuca & W. Stuerzlinger (2019) | $MT = a + b(ID) + c(CTD)$ | 3.2 |
| 5 | J. Accot & S. Zhai (2003) | $MT = a + b\, log\left(\sqrt{\left(\frac{A}{W}\right)^2 + \alpha_1\left(\frac{A}{H}\right)^2} + 1\right)$ | <2.0 |
| 6 | E.R. Hoffmann (1995) | $MT = a + b\, log_2\left(\frac{2R}{S+P}\right)$ | <2.0 |
| 7 | Two-part (Welford, 1968) | $MT = a + b_1\, log(A) - b_2\, log(W)$ | <2.0 |
| 8 | S. MacKenzie & W. Buxton (1992) | $MT_{min} = a + b\, log_2\left(\frac{A}{min\,(W_1 \propto_1 H)} + 1\right)$ | <2.0 |
| 9 | D.E. Meyer *et al.* (1988) | $MT = a + b\sqrt{\frac{A}{W}}$ | <2.0 |
| 10 | A. Murata & H. Iwase (2001) | $MT = a + b(sin\,\varphi) + c\, log_2\left(\frac{A}{W} + 1\right)$ | <2.0 |
| 11 | Y. Cha & R. Myung (2013) | $MT = a + b\varphi + c\, sin(\theta) + d \cdot log_2\left(\frac{2D}{W+F}\right)$ | <2.0 |
| 12 | Shannon-Welford(Shoemaker *et al.*, 2012) | $MT = a + b_1\, log(A + W) - b_2\, log(W)$ | <2.0 |

**Source:** developed by the authors based on M. Amini *et al.* (2025)

Despite cases of indirect tangency of individual variations to three-dimensional adaptation, such studies were indicated as full-fledged models, or serve as a model-creating basis. For example, research by L.D. Clark *et al.* (2020) covered equations 4, 6, 8, 11 (Table 4) to identify potential factors affecting *MT* in VR. I. Janzen *et al.* (2016) equations 2, 7, and 12 (Table 4) were studied to analyse the effect of target depth on performance in target-designating tasks. D. Yu *et al.* (2019), used the Shannon equation (MacKenzie, 1992), but the basic calculations were based on the experiment of R.W. Soukoreff & I.S. MacKenzie (2004), which can serve as partial proof of the greater effectiveness of these results. About a fifth of the studies show the use of Fitts' law alone, without specifying the selected variations used in the calculations, which makes it difficult to estimate them, since there is no indication of whether the accuracy adjustments were adapted. Despite this, it is the work to improve situational efficiency and the search for universality that are the most popular cases of using variations of Fitts' law.

In three-dimensional environments, the primary equation of Fitts' law (equation 3) shows lower citation rates. The classical interpretation is found mainly in relatively outdated works. Thus, for example, B. Watson *et al*. (2003) used the classical calculation of the *ID* instead of the *MT* equation. It is worth mentioning the material by N. Li *et al*. (2020), which briefly contains the classical formulation without signs of further involvement in the methodology. Such a case is considered isolated, so its percentage share is insignificant. In the course of literature analysis by D. Wolf *et al*. (2018) was found that only the *IP*, i.e., the inverse slope value, was used to calculate the transmittance (*TP*). It was also found that the $TP = ID/MT$ equation was mostly used to calculate TP instead of the typical slope-based calculation ($TP = 1/b$). All the above exceptions and alternative interpretations emphasise the need for a clear classification and systematisation of existing adaptations. The development of this segment directly depends on the scientific level of the sphere of spatial computers, considering such factors as the depth of visibility, the level of immersion of the user in virtual space and, accordingly, their behaviour models.

The differences between calculation equations identified in the course of the study and the corresponding results of their statistical distribution were explained primarily by significant chronological gaps between experiments during which research changed area or expanded the scope of conditions due to the actualisation of Fitts' law in the development of increasingly complex human-machine interaction interfaces. The difference in accounting for additional factors shown in Table 2 is a key factor demonstrating why the effort by A.T. Welford (1960) and I.S. MacKenzie (1989) improved the primary results, and the study by E.R. Hoffmann (1995) aimed to expand the theoretical foundation with the possibility of further spatial development. The selection of adaptations from Table 3 reflects the difference in approaches when choosing additional parameters of the three-dimensional environment (depth or distance to the object, its dimensions, the width of the user's finger, etc.), which have a significant impact on the accuracy of the results of travel time forecasts, and their coordination with the results of P.M. Fitts (1954) for further three-dimensional research (Batmaz *et al*., 2023). The results obtained are an example of how a single equation with the participation of several groups of authors was adapted to different levels of interaction.

The advantages and relevance of the selected solutions are explained by their focus on developing spatial interfaces, especially for VR and AR. A representative example is the Mackenzie equation, which is universal under two-dimensional conditions (Welford, 1960), but does not stand up to the test of "time and space", that is, it loses its relevance with the development of three-dimensional research. According to Table 4, complicating the equation with as many parameters as possible does not necessarily increase its effectiveness (Hoffmann, 1995; Murata & Iwase, 2001; Cha & Myung, 2013), and the possibility of practical testing using VR and/or AR tools significantly speeds up the course of research (Batmaz *et al*., 2023). Systematisation of equations, their author's variations (tables 2-4) and areas of application (Table 1) determined the scenario, conditions and equipment for using each of the adaptations of Fitts' law.

One example of practical implementation is the study of motor behaviour in extended reality (XR) environments. The paper by M. Amini *et al*. (2025) noted that the classical model demonstrates efficiency in calculating pointing tasks, but for more complex three-dimensional scenarios, it requires expansion of spatial parameters. The experiment considers the depth indicators to the target, the features of the input means (beam pointer, virtual limb model), and the variability of algorithms for determining spatial coordinates. With this approach, the equation loses its versatility, but forms a number of more accurate models of user behaviour under certain conditions.

The study by H. Xiao *et al*. (2024) modified model by A. Murata & H. Iwase (2001) using a separate angular parameter using a trigonometric function to determine the direction of movement, while simultaneously complementing the classical equation with a dynamic depth parameter based on the study by M.D. Barrera Machuca & W. Stuerzlinger (2019), which is especially important for VR environments. Combining the experience of colleagues allowed the researchers to improve the accuracy of predicting the time of movement in space, based on the vector orientation and spatial characteristics of the scene. However, the complexity of the model remains relevant and effective only for a specific use case. This technique is supplemented in the study by Y. Cha & R. Myung (2013), where among the auxiliary variables in the equation, the parameter of the effective contact width on a three-dimensional plane is considered. Under such conditions, the *ID* more accurately coincides with the actual indicators in real conditions of using VR tools. The need for these solutions is due to differences not only in space conditions, but also in the corresponding equipment, since different types of manipulators and reading tools (controllers, gesture trackers) have different effects on *MT*, which corresponds to the conclusions of previous studies.

## Conclusions

As a result of searching for publications in databases, their analysis and systematisation, a representative sample of scientific papers was formed, united by the goal of developing theoretical models based on Fitts' law for modelling user sensory-motor behaviour in two-dimensional and three-dimensional interfaces. Using the formalised dependencies of *ID* and *MT*, in combination with an overview of the available results of practical tests, a selection of studies was made based on the relevance, completeness and reliability of modelling. It was found that the number of citations of Fitts' law in publications during 2000-2023 is constantly increasing, and the most common area of research is interaction and evaluation of the system (62.3%). The advantages of the obtained result are verification not only by index indicators, but also by the correspondence of theoretical equalities with formalised experimental data. This approach helped to eliminate methodological contradictions and avoid the fragmentary use of equations inherent in review articles. The accuracy of the obtained result was explained by the use of quantitative selection criteria and the consistency of values that ensure mathematical equality in all versions of adaptations.

Theoretical analysis of the effectiveness of adaptations of Fitts' law has demonstrated that the two-dimensional equation by I.S. MacKenzie best complemented his experience in modelling on a two-dimensional plane and achieved the highest accuracy of calculations, which was confirmed by minimal deviations in the results from those calculated using equation (2). Simultaneously, it did not have high citation rates among three-dimensional adaptations (< 2%). However, according to the results of statistical analysis, the most cited adaptation of Fitts' law in the three-dimensional plane is the model by Shannon with an indicator of 77.8%, and the second most cited model is the angular model with an indicator of 28.7%. All other studies, including the original paper by P.M. Fitts (4%), have an equally low citation rate. The result obtained is explained by the lack of a standardised approach to practical verification of results, differences in research areas and chronological differences in hardware development, since the original equation was published back in 1954, when the development of three-dimensional interfaces almost did not exist.

The study has a number of limitations that may affect the results of practical tests. Firstly, only equations available from open sources were considered, so the impact of hidden successes in the private and commercial development sectors is sometimes impossible to assess until the release of the corresponding product. Secondly, the difference in experimental contexts and research areas makes it difficult to directly compare the results. Thirdly, not all models shown in Tables 1-4 were tested under the same hardware conditions, which affects the reproducibility of results in real AR and VR systems. The disadvantages of the study are that, unlike Mackenzie's consensus-universal equation for two-dimensional interaction, even the most cited equations are only theoretical variations without definitive general recognition, which was noted by other researchers. In addition, even at this stage, a significant number of simplifications are allowed, which will only be eliminated if significant success is achieved at this stage, before drawing conclusions from real-world interaction scenarios. Further development of the research involves updating the statistical distribution considering new papers from open sources, expanding the experimental base, and unifying the testing methodology, and testing theoretical examples on real hardware. This combination of methods will speed up the reproduction of theoretical models in real-world user scenarios, making it possible to include the factors that have not been considered in influencing the results, thereby more clearly defining the limitations when applying adaptations of Fitts' law.

## Acknowledgements
None.

## Funding
None.

## Conflict of Interest
None.

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# Адаптація закону Фіттса в тривимірних інтерфейсах

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**Анотація.** Відсутність консенсусного вибору розрахункової моделі адаптації закону Фіттса при розробці тривимірних інтерфейсів, а також неоднорідність методології досліджень з відповідної тематики ускладнює зпівставлення результатів між собою. Метою дослідження було визначення найбільш ефективної розрахункової моделі користувацького досвіду при розробці просторових інтерфейсів у засобах віртуальної та реальності в умовах двовимірних та тривимірних середовищ. Для цього було проведено метааналіз репрезентативної вибірки наукових праць, розподілених за критеріями виміру (одновимірні, двовимірні, тривимірні) та напрямками досліджень, а також впорядкованих за відсотком цитувань. За результатами формалізованих критеріїв розподілу, найвищий показник цитованості у тривимірних працях мало рівняння Шеннон, запропоноване Маккензі (77.8 %) та кутове рівняння Мачуки (28.7 %). Первинна формула Фіттса в цьому контексті, навіть як першоджерело мала 4 % актуальності, а найбільш ефективне рішення в умовах двовимірної площини за авторства Маккензі – лише 2 %. Виявлено, що найбільш релевантними є тривимірні моделі із залученням додаткових параметрів, серед яких тригонометричні кутові коефіцієнти та набори координат для визначення глибини або складних траекторій досягнення цілей, що підтверджено високими показниками цитованості відповідних підходів. Встановлено, що ефективність кожної з продемонстрованих адаптацій закону Фіттса не визначається лише за типом та кількістю підібраних параметрів, оскільки не менш важливу роль відіграє процес порівняння теоретичних розрахунків з емпіричними даними, отриманими в результаті взаємодії користувачів з апаратними коплексами, динаміка доступності яких для широкого кола дослідників кардинально змінювалась впроводж невеликого проміжку часу (10-20 років), враховуючи загальний хронологічний діапазон розглянутих праць (1954-2025 рр.). Практична цінність дослідження полягає в тому, що отримані результати можна використовувати для розробки інтерфейсів користувача в засобах віртуальної та доповненої реальності, формування стандартів та методичних рекомендацій при роботі з просторовими інтерфейсами, покращення зворотнього зв'язку при аналізі користувацького досвіду

**Ключові слова:** просторові інтерфейси; віртуальна реальність; доповнена реальність; інтерфейс користувача; сенсорно-моторна поведінка